

The snow puzzle

Some years ago a friend and I did a kayak trip from Makara, near Wellington, at the bottom of the west coast of New Zealand's North Island. This small settlement is a common departure point for people crossing Cook Strait by kayak and has great views across to the South Island. The channel at this narrowest



point is about 33km (about 20 miles) across, and the part of South Island we could see is a complex area of sounds separated by steep ridges, with peaks up to 1200m (3,900 ft).

It was June (mid-winter in the southern hemisphere), and the clearest, crispest, coldest sort of day. The views of the South Island were stunning. Cold weather generally offers better visibility as the air cannot hold so much humidity. While marvelling at the view we were puzzled to see freshly fallen snow

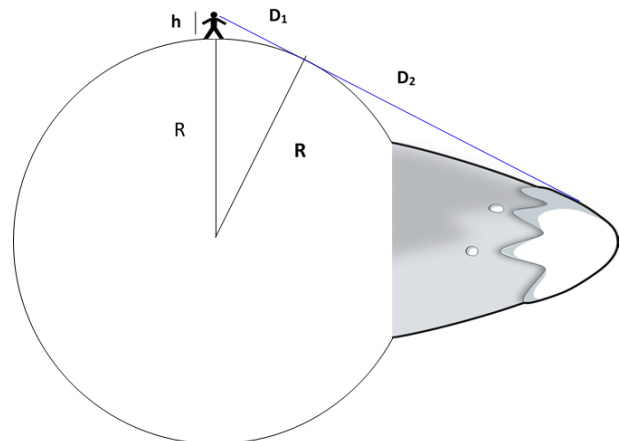
right down to sea level on the other side. It had been chilly it's true, but it seemed very odd that the snow would come all the way to the shore line on one side of the strait with no sign at all of it where we were.

Musing on this, I wondered if the curvature of the earth might obscure the lower slopes of the distant hills. I recalled being told as a lad that to a six foot tall man standing on the shore, the horizon was three miles (4.8 km) away. I put this puzzle in my mental filing cabinet and only recently found it again. In the meantime, Google had come along to solve most our personal problems.

<http://blogs.discovermagazine.com/badastronomy/2009/01/15/how-far-away-is-the-horizon/#.V01Wt5F97IU> shows a similar

representation to this of a person standing on the surface of the earth with d_1 as the distance to the horizon.

It also shows the calculation $d_1^2 + R^2 = (R + h)^2$ where R , is the earth's radius and d_1 is the distance to the horizon. It gives a table that shows the eye's height above sea level and the distance to the horizon. The lowest height the author gives is 1 metre. However it was easy to replicate the spreadsheet for lower heights, as a person seated in a kayak is not a metre tall. Sitting on the floor revealed my eye height to be about 75 cm (29½ in) above 'sea level'. From that height, the horizon is 3.09 km (1.92 miles) away.



I think that the line of sight continues straight on, and that a point well up the mountain appears to be at sea level as shown in the diagram. D_2 is the distance from the same point on the real horizon to the point that appears to be the horizon where the line of sight hits land.

Metric		Imperial	
Eye height above sea level (m)	Distance to horizon (km)	Eye height above sea level (in/ft)	Distance to horizon (miles)
0.65	2.88	24	1.73
0.70	2.99	26	1.80
0.75	3.09	28	1.87
0.8	3.19	30	1.94
0.85	3.29	33	2.03
0.9	3.38	3	2.12
1	3.57	6	3.00
2	5.05	9	3.67
5	7.98	12	4.24
6	8.74	15	4.74
7	9.44	18	5.19
8	10.09	21	5.61
9	10.70	24	6.00
10	11.28	27	6.36
20	15.96	30	6.70
50	25.23	60	9.48
100	35.68	100	12.24
200	50.46	200	17.31
500	79.78	500	27.37
1000	112.83	1000	38.71
10000	356.93	10000	122.41

This modified table shows a variety of eye heights and the corresponding distance to the horizon

I believe the table can also be used backwards. If a person could see the horizon 50km away how high above sea level would they be? By good fortune the table has a distance to horizon of 50.46 km and the table shows that the theoretical eye would be at an altitude of 200m. Likewise from the Imperial measurement side of the table, if the horizon was 38.71 miles away, the eye height must be 1,000 feet above sea level.

So from our kayaks, what appeared to be the shore line on the South Island could have been at an actual altitude of 200 metres above sea level if we were looking at a point say 50 km away.

Using this method, for a kayak paddler on a calm sea with an eye height is 28 inches, the horizon is 1.87 miles away. If the point viewed is 1000 ft above sea level, it is a further 38.71 miles past the horizon, giving a total distance of 40.58 miles.

If the eye height is 75 cm, the horizon is 3.09km away. If the point viewed is 500m above sea level, it is a further 79.78 km from that point ie a total of 82.87km away. In New Zealand's clear air it is no problem at all to see mountains much further away than this distance.

The original rule of thumb that a 6 foot man on the shoreline sees a sea horizon 3 miles away is remarkably accurate, except that he'd need to be significantly taller so his eyes were at 6ft altitude. If any pinnipeds wish to try this experiment they should sit exactly on the shoreline with their eyes at seal level.